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$$z_1 = \left(\frac{-1 \pm \sqrt{(1 - \frac{2^{5.6}}{2^7} a^6)}}{2} \right)^{\frac{1}{3}} + \frac{4a^2 \sqrt[3]{2}}{[-1 \pm \sqrt{(1 - \frac{2^{5.6}}{2^7} a^6)}]^{\frac{1}{3}}};$$

$$z_2 = w \left(\frac{-1 \pm \sqrt{(1 - \frac{2^{5.6}}{2^7} a^6)}}{2} \right)^{\frac{1}{3}} + \frac{4a^2 \sqrt[3]{2}}{w[-1 \pm \sqrt{(1 - \frac{2^{5.6}}{2^7} a^6)}]^{\frac{1}{3}}};$$

$$z_3 = w^2 \left(\frac{-1 \pm \sqrt{(1 - \frac{2^{5.6}}{2^7} a^6)}}{2} \right)^{\frac{1}{3}} + \frac{4a^2 \sqrt[3]{2}}{w^2[-1 \pm \sqrt{(1 - \frac{2^{5.6}}{2^7} a^6)}]^{\frac{1}{3}}}.$$

$\therefore y_1 = z_1 - 2a/3$, $y_2 = z_2 - 2a/3$, and $y_3 = z_3 - 2a/3$, are the three required values of y ; and $x_1 = 1/y_1^2$, $x_2 = 1/y_2^2$, $x_3 = 1/y_3^2$, are the three values of x . It is easily seen that the values of y , and hence of x , are all real, one real and two imaginary, or real and multiple, according as $a^6 >$, $<$, or $= \frac{2^{7.6}}{2^8}$. [DRANE.]

GEOMETRY.

122. Proposed by G. I. HOPKINS, A. M., Professor of Mathematics and Physics, Manchester High School, Manchester, N. H.

If perpendiculars are dropped from the vertices of a regular polygon upon any diameter of the circumscribed circle, the sum of the perpendiculars which fall on one side of this diameter is equal to the sum of those which fall on the opposite side. [From Chauvenet's *Treatise on Elementary Geometry*.]

III. Solution by WILLIAM H. ECHOLS, B. S., C. E., Professor of Mathematics in University of Virginia, Charlottesville, Va.

A very simple proof of this is:

1°. Let there be a regular polygon of $2n$ sides, $p_1, \dots, p_{2n}$, the perpendiculars from vertices on any straight line.

Then if p_c is perpendicular from the center, clearly

$$\sum_1^{2n} p_r = 2np_c.$$

2°. Form a polygon of n sides by joining the mid-points of alternate sides of 1°. Then at once

$$2 \sum_1^n p'_r = \sum_1^{2n} p_r = 2np_c,$$

where p'_r are perpendiculars of n -side.

This is true whatever integer be n .

3°. If the line of projection passes through the center, the theorem is demonstrated generally.

125. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

To find the locus of a point on the surface of an ellipsoid which has the property that the tangent plane at that point is at the given distance, f , from the center of the ellipsoid.

Solution by **WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.**

The equation to the tangent plane to the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \dots\dots (1)$$

at the point (x', y', z') is

$$\frac{x'x}{a^2} + \frac{y'y}{b^2} + \frac{z'z}{c^2} = 1 \dots\dots (2).$$

Then

$$\frac{1}{f^2} = \frac{x'^2}{a^4} + \frac{y'^2}{b^4} + \frac{z'^2}{c^4} \dots\dots (3),$$

is the required locus, an ellipsoid concentric with (1).

Also solved by **G. B. M. ZERR**, and **ELMER SCHUYLER**.

126. Proposed by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

Through any fixed point O draw two straight lines at right angles. Let one line cut a given circle at Q , the other at R . Find, by Euclidean methods, the locus of the foot of the perpendicular from O upon the chord QR . Give complete analysis and discussion. Solve also by coördinate geometry.

I. Solution (Analytical) by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Let r = the radius of the given circle, a = the distance of its center from O , and take the line through O and the center for the axis of x . Then the coördinate axes being rectangular, the equation to the fixed circle is

$$(x-a)^2 + y^2 = r^2 \text{ or } x^2 + y^2 - 2ax + a^2 - r^2 = 0 \dots\dots (1).$$

If $lx + my = 1 \dots\dots (2)$ be the equation to QR , the equation to the lines OQ and OR is

$$x^2 + y^2 - 2ax(lx + my) + (a^2 - r^2)(lx + my)^2 = 0 \dots\dots (3), \text{ or}$$

$$[1 - 2al + (a^2 - r^2)l^2]x^2 + 2[lm(a^2 - r^2) - am]xy + [1 + (a^2 - r^2)m^2]y^2 = 0 \dots\dots (4).$$

The condition that these lines are perpendicular to each other is

$$2 - 2al + (a^2 - r^2)(l^2 + m^2) = 0 \dots\dots (5).$$

The line through O and perpendicular to (2) is $x/y = l/m \dots\dots (6).$

Making (5) homogeneous by aid of (2), we have,

$$[2x^2 - 2ax + (a^2 - r^2)]l^2 + (4xy - 2ay)lm + [2y^2 + (a^2 - r^2)]m^2 = 0 \dots\dots (7).$$